Performance Analysis and Adaptive Power Control for Block Coded Collaborative Networks [*]

W. Guo, I. Chatzigeorgiou, I. J. Wassell
Digital Technology Group, Computer Laboratory
University of Cambridge, UK
wg209@cam.ac.uk

R. Carrasco
Department of EE&C Engineering
University of Newcastle, UK
r.carrasco@ncl.ac.uk

ABSTRACT
We derive theoretical bit and frame error rate expressions for decode-and-forward (DF) collaborative networks containing M users, employing a variety of block codes over a Rayleigh block faded channel. With the aid of these expressions, we explore the performance of adaptive power control for such systems. This extends previous work by optimizing power allocation for all relay and direct channels. We further extend our work to a variety of cooperation mechanisms and we conclude that power control can greatly benefit a DF collaborative network in a fading environment.

Categories and Subject Descriptors
H.4 [**Information Systems Applications**]: Miscellaneous

General Terms
Collaborative Network

Keywords
Collaborative, Power Allocation, Block Codes, Diversity

1. INTRODUCTION
Collaborative networks involve users cooperating with each other by sharing information and transmitting each other's data to the destination [6]. Diversity arising from the use of different fading paths can improve system performance. We will utilize this cooperative scheme in the context of a fixed wireless access (FWA) network. The motivation for this paper is to propose a power control scheme, which relies on the use of channel state information at the transmitter (CSIT), and compare its performance to that of other schemes including equal power allocation, which requires no CSIT. We derive theoretical bit error rate (BER) and frame error rate (FER) expressions for a variety of block codes in a decode-and-forward (DF) collaboration system [9]. Simulations are used to verify the accuracy of our expressions.

Power allocation schemes exist in the form of water-filling algorithms for amplify-and-forward (AF) cooperative networks [14], [3], [11], as well as networks employing coded cooperation [5]. Whilst there has been work to solve power allocation for DF networks in both fading and Gaussian channels [8], complexity considerations have limited the optimization to only the relay-to-destination paths [7]. Therefore, we believe that power allocation for DF networks under block fading remains a challenge. We propose a method appropriate for block coded DF cooperative networks that optimizes power allocation in all the channels involved, under a longer term power constraint than [7].

We first derive theoretical BER expressions of block codes under a block faded DF collaborative environment. After which, we determine both an adaptive and deterministic optimized power allocation for the system. Our scheme is then compared to other cited allocation methods. We also investigate various collaborative network schemes, whereby users can act cooperatively for mutual benefit or selfishly and leech the shared resources. The performance of each scheme and how influential power allocation is on each, is analyzed.

2. PRELIMINARIES
Transmission is assumed to take place over an additive white Gaussian noise (AWGN) block fading channel. This is appropriate for a slowly changing environment such as that experienced in a FWA system. Each data block has a size equal the length of the block code used, n. Within a block period, the channel gain coefficient remains constant, and any power adaption occurs between the blocks. In our system we will refer to the channel linking a user to another user as the *interuser channel*, and the channel linking a user to the destination as the *uplink channel*. We define γ as the instantaneous signal to noise ratio (SNR) of the uplink channel, $\overline{\gamma}$ as the channel's average SNR, and $\overline{\gamma}_R$ is the average inter-user channel SNR.

2.1 Error Rate Expressions
The block codes we have chosen to use are perfect codes [1]. Indeed we have chosen the (7,4) Hamming code to demonstrate system performance, unless otherwise stated. Our approach yields an upper bound performance to non-perfect block codes. Perfect codes can correct a maximum of E errors:

$$E = \lfloor \frac{d_{min} - 1}{2} \rfloor, \qquad (1)$$

where d_{min} is the minimum Hamming distance. The BER and FER for any perfect block code in a block fading channel are given by [10]:

$$BER = \frac{1}{n} \sum_{i=E+1}^{n} \binom{n}{i} \lambda_i p^i (1-p)^{n-i}, \qquad (2)$$

$$FER = \sum_{i=E+1}^{n} \binom{n}{i} p^i (1-p)^{n-i}, \qquad (3)$$

where λ_i denotes the average number of symbol errors remaining in the corrected sequence given that the channel caused i symbol errors; the value of λ_i for various types of block coding can be determined using look up tables [10]. Note that a common approximation uses $\lambda_i = i$.

2.2 Erfc Approximation

When M direct channels having the same average SNR value $\overline{\gamma}$ are decoded at the destination using maximum ratio combining (MRC), the resultant symbol error probability (p_{MRC}) is given by [12] and [4]:

$$p_{\mathrm{MRC}} = \int_0^\infty \frac{\gamma^{M-1}}{\overline{\gamma}^M (M-1)!} \exp\left(-\frac{\gamma}{\overline{\gamma}}\right) \frac{1}{2} \mathrm{erfc}(\sqrt{\gamma}) \mathrm{d}\gamma. \qquad (4)$$

In the case of $M = 1$, we obtain the expression for a direct channel's symbol error probability (p_{direct}). The complementary error function $\mathrm{erfc}(z)$ is defined as:

$$\mathrm{erfc}(z) \equiv \frac{2}{\sqrt{\pi}} \int_z^\infty \exp\left(-t^2\right) dt. \qquad (5)$$

Equations (2) and (4) are central to deriving an approximation for the BER of block codes in both direct as well as collaborative channels. However, when (5) is substituted into the BER (2) and FER (3) expressions, which contain n^{th} order polynomials, the final expression does not have a closed form solution. We therefore choose to approximate the $\mathrm{erfc}(z)$ function. There are many existing approximations of the function, of which most are derived from series expansions of the original or a sinusoidal variation [2]. Whilst there is a good single term upper bound [13], we will use the following single term exponential approximation:

$$\mathrm{erfc}(z) \approx 0.8 \exp\left(-1.8z^2\right), \qquad (6)$$

which was derived by minimizing the error over medium to high SNR values. The single term exponential nature of the approximation allows us to simplify the block code BER expressions. Subsequently the probability of an un-coded symbol error of an MRC channel is approximated to:

$$p_{\mathrm{MRC}} \simeq \frac{0.4}{(1 + 1.8\overline{\gamma})^M}. \qquad (7)$$

2.3 Case Study: Hamming Codes

Although accurate expressions exist for all perfect block codes with known λ_i values, we shall demonstrate our approach using the (7,4) Hamming Code. The equations listed are generic approximations for all block codes, and our analysis approach can be extended to other block codes and act as a lower bound for non-perfect block codes. The implications for other block codes will be discussed later on. By substituting the approximation (7) into (2) and (3), we are able to derive closed form BER and FER expressions for

Hamming codes over a direct point to point as well as a MRC channel. For the MRC of M channels at the destination, the BER expression is:

$$BER_{\mathrm{MRC}} \simeq \frac{1}{n} \sum_{i=E+1}^{n} \sum_{k=0}^{n-i} \binom{n}{i} \binom{n-i}{k} \frac{i(-1)^k (0.4)^{k+i}}{(1 + 1.8\overline{\gamma}(i+k))^M}, \qquad (8)$$

where the λ_i in (2) has been approximated to i. In the case of $M = 1$, the BER and FER of a direct channel are:

$$BER_{\mathrm{Direct}} \simeq \frac{1}{n} \sum_{i=E+1}^{n} \sum_{k=0}^{n-i} \binom{n}{i} \binom{n-i}{k} \frac{i(-1)^k (0.4)^{k+i}}{1 + 1.8\overline{\gamma}(i+k)}, \qquad (9)$$

and

$$FER_{\mathrm{Direct}} \simeq \sum_{i=E+1}^{n} \sum_{k=0}^{n-i} \binom{n}{i} \binom{n-i}{k} \frac{(-1)^k (0.4)^{k+i}}{1 + 1.8\overline{\gamma}(i+k)}. \qquad (10)$$

The exact expressions: (2), and (3) are obtainable via numerical integration. They provide almost identical matches to simulation results. However, (10) also yields results close to those of the simulation, yet it has a form that makes it amendable for the development of a power allocation algorithm and other useful expressions.

3. DF COLLABORATIVE NETWORKS

In a collaborative network, two users try to transmit independent information to the destination, using each other as their relays. We shall derive expressions for a M sized network, but demonstrate with simulations for 2 or 3 users cooperating. Fig. 1 shows a collaborative network, involving two users. We will show that the system user's BER comprises of two possibilities: i.e., that when cooperating and that when not cooperating. For the two possibilities we define the following power allocation factors: α is the power allocation factor during a non-cooperative stage, and β that during a broadcasting or cooperative stage. The factors α and β are illustrated in Fig. 1 and in the case of equal power allocation are both set to 0.5. In the first time block, the users both broadcast to each other as well as to their destination at the same power. In the second time block, they act as relays for each other and require a successful decoding of the message before forwarding it to the destination. Users may choose to cooperate despite no cooperation by the other partner, or form a consensus whereby cooperation only occurs mutually. We will show the user receiving cooperation will benefit greatly, whilst the other user will suffer a halved uplink power for its own data. There are different cooperation subtypes, which we will now explore.

3.1 Unselfish Cooperation

First we look at unselfish cooperation between M users, whereby cooperation occurs when possible, irrespective of the other users's action. Fig. 1 shows the system diagram for a two-user DF system. In the first stage all users broadcast their data to each other and to the destination. Since the users operate unselfishly independent to each other, the BER of the system is the same as the BER for each user. In the second stage, there are the following possible scenarios with respect to a user: all, some or none of the other users cooperate. Therefore the BER of a user can be obtained as
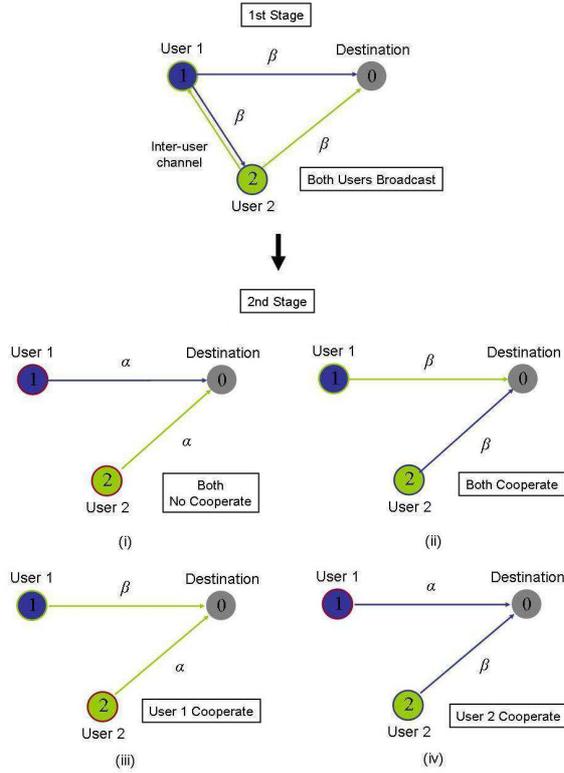
Figure 1: Unselfish Collaboration for $M = 2$ Users

follows:

$$BER_{\text{Unselfish}} = \frac{1}{n} \sum_{i=E+1}^{n} \sum_{k=0}^{n-i} \sum_{j=0}^{M-1} \binom{n}{i} \binom{n-i}{k} \binom{M-1}{j} \\ P_C^j (1-P_C)^{M-1-j} \frac{i(-1)^k (0.4)^{k+i}}{(1 + 1.8\overline{\gamma}(i+k))^{j+1}}, \tag{11}$$

where the chance of cooperation is $P_C = 1 - FER_{\text{Direct}}$. The expressions for each of the terms are given by equations (8), (9) and (10).

3.2 Mutually Selfish Cooperation

In a more realistic case, we examine the case where each user will only cooperate when both sides can mutually cooperate. We call this *mutually selfish cooperation*. Therefore, if any user fails to successfully decode the other user's data, no cooperation occurs for either side, and subsequently normal direct transmission occurs. Only scenarios (i) and (ii) in Fig. 1 are possible in the second stage. This way, there is either mutual benefit, or no cooperation. Hence the chance of cooperation is now P_C^2 for $M = 2$. Comparing the optimal unselfish system and the more realistic mutually selfish scheme: there is no difference when either the *interuser channel* is perfect, or non-existent. When the *interuser channel* is perfect: cooperation is always possible; and when the *interuser channel* is non-existent: cooperation isn't possible irrespective of the scheme. This is mirrored in the mutually selfish

BER theoretical expressions:

$$BER_{\text{Selfish}} = \frac{1}{n} \sum_{i=E+1}^{n} \sum_{k=0}^{n-i} \sum_{j=0}^{M-1} \binom{n}{i} \binom{n-i}{k} \binom{M-1}{j} \\ P_C^{2j} (1-P_C^2)^{M-1-j} \frac{i(-1)^k (0.4)^{k+i}}{(1 + 1.8\overline{\gamma}(i+k))^{j+1}}. \tag{12}$$

The difference in the expressions (12) and (11) for $M = 2$ can be simplified to:

$$BER_{\text{Difference}} = BER_{\text{Selfish}} - BER_{\text{Unselfish}} \\ = P_C(1 - P_C)(BER_{\text{Direct}} - BER_{\text{MRC}}). \tag{13}$$

From (13), we can make two observations. The first is: given a fixed interuser SNR, the BER difference increases as we increase the uplink SNR. The second is: given a fixed uplink SNR, the maximum difference occurs when $P_C = 0.5$.

3.3 Cooperative User and Selfish Leecher

We explore the case when some users cooperate unselfishly, and another user chooses to be a selfish leecher. We define a leecher as a user that seeks cooperation from other users, but never relay anyone else's data. From simulations (not shown), the selfish leeching user gains a greater benefit by never cooperating and relies on the other *unaware* users to cooperate whenever they can, whilst the cooperating users suffer a degradation in performance. Using a similar analysis, the BER difference between this case and unselfish cooperation for $M = 2$ users is:

$$BER_{\text{Difference}} = P_C \times (BER_{\text{Direct}} - BER_{\text{MRC}}). \tag{14}$$

This shows that the leeching user benefits the most when the chance of cooperation is at a maximum (high interuser SNR).

3.4 Discussion

Unselfish cooperation is the ideal cooperative scenario in terms of maximizing the system's performance independent of channel parameters, and we show why this is the case with the aid of (13). However, given non-symmetric and selfish behavior scenarios, one user will inevitably benefit more at another's expense. We have shown the exact BER difference between an optimal unselfish network over that of a more realistic, but mutually selfish one. Also, we found how a selfish leeching node's benefit is proportional to the SNR of its *interuser channel*. Each finding is backed up by simulations (not shown) as well as theoretical approximations. We found that codes performed independently of the system configuration, and note that the expressions we obtained are lower bounds for non-perfect block codes. All of the above work was extended to Golay codes, and Reed Solomon codes, and multi-node networks.

4. POWER ALLOCATION

For power allocation to be fair and comparable to other schemes, we add a power constraint whereby the amount of power available to each user is fixed over a period of time (N cooperation blocks). The specific value of N only needs to be large enough to statistically describe the probability of cooperation (P_C) with reasonable accuracy. The overall average power is defined to be unity, allowing a fair comparison to be made with an equal power allocation scheme. In

an unselfish collaborative network as described previously, the power constraint is divided into two stages per cooperation block. Referring to Fig. 1: the first stage will broadcast at power β, and the second stage will transmit at power α with probability $1 - P_C$ and at power β with probability P_C. Therefore, the power constraint for an unselfish network is:

$$\alpha(1 - P_C)^{M-1} + \beta\left(1 + \sum_{j=1}^{M-1} jP_C^j(1 - P_C)^{M-1-j}\right) = 1. \quad (15)$$

The BER expression is shown in (11). We introduce power allocation factors to the expression:

$$\begin{aligned}
BER_{\text{Unselfish}} = &\frac{1}{n} \sum_{i=E+1}^{n} \sum_{k=0}^{n-i} \binom{n}{i}\binom{n-i}{k} \sum_{j=1}^{M-1} \binom{M-1}{j} \\
&P_C^j(1 - P_C)^{M-1-j} \frac{i(-1)^k(0.4)^{k+i}}{(1 + 1.8\overline{\gamma}(i+k)\beta)^{j+1}} \\
&+ (1 - P_C)^{M-1} \frac{i(-1)^k(0.4)^{k+i}}{1 + 1.8\overline{\gamma}(i+k)\alpha}.
\end{aligned}$$
$$(16)$$

Note that P_C is derived from equation (10), and for unselfish cooperation it is: $1 - FER_{\text{Direct}}$, i.e.,

$$P_C \simeq 1 - \sum_{i=E+1}^{n} \sum_{k=0}^{n-i} \binom{n}{i}\binom{n-i}{k} \frac{(-1)^k(0.4)^{k+i}}{1 + 1.8\overline{\gamma_R}(i+k)\beta} \quad (17)$$

where $\overline{\gamma_R}$ is the *interuser channel* average SNR. This is the same expression as introduced in (10), with the addition of the power allocation factor β due to the previously mentioned broadcast nature of the source in the first stage. Therefore, the power allocation not only affects the uplink SNR, but also the *interuser channel* SNR (i.e., the chance of cooperation, P_C).

4.1 Iterative Power Allocation Algorithm

We first utilize a brute force search approach, which iteratively searches along all valid FER possibilities, subject to the power control constraint (15). The process of power allocation begins with incrementing β from 0 to *limit* and then using this value to find the corresponding α and P_C from (15) and (17). The *limit* is determined by the value of β for which $\alpha \geq 0$. For each β realization, we calculate the FER using (16). The lowest FER and its corresponding power allocation factors are stored and subsequently applied to the simulation to verify our scheme.

4.2 Deterministic Power Allocation

Instead of optimizing the factors through the aforementioned search algorithm, a delay-less deterministic method is now introduced. We can differentiate the BER expression (16) with respect to either α or β (since the two power allocation factors are related by the power constraint (15)). This reveals a power allocation factor without using the optimization described earlier, but suffers a slight loss of accuracy due to the approximations made. We see it still outperforms the equal power allocation scheme. For the differentiation, we use the binomial approximation with Bernoulli's inequality: $(1-p)^n \geq (1-np)$. This is accurate if p is small and less than 1. In our case, p is the probability of a single bit error, which is small and can never exceed 1. By differentiating the BER
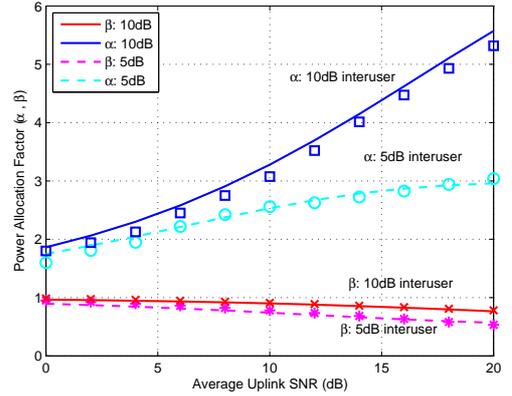
Figure 2: Effect of Uplink and Interuser Channel Average SNRs on the Power Allocation Factors of $M = 2$ users: symbols indicate simulation results and lines indicate theoretical expressions.
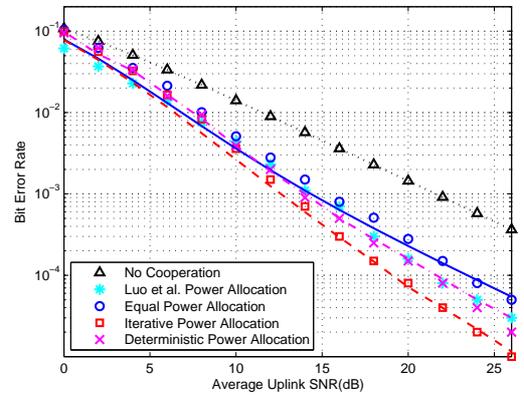
Figure 3: Unselfish Collaboration in 10dB Interuser Average SNR Environment for $M = 3$ users: symbols indicate simulation results and lines indicate theoretical expressions.

with respect to β, we find the approximated expression for β which gives the minimum BER:

$$\beta \simeq 1 - \sqrt{\overline{\gamma}}\frac{\sqrt{\overline{\gamma_R}^2 + \overline{\gamma}}}{\overline{\gamma_R}^2 - \overline{\gamma}}. \quad (18)$$

This is consistent with our observations. From Fig. 2 we note that the power allocation factor given to cooperation β, increases as the *interuser channel* ($\overline{\gamma_R}$) improves. However, β declines as the uplink channel ($\overline{\gamma}$) improves, due to more power given to α. The increase in α as the *uplink channel* improves is expected as a better uplink SNR favors more direct transmission. As the *interuser channel* SNR improves, the occurrence of non-cooperative direct transmission decreases, allowing more power allocation to direct transmission, whilst maintaining the power budget.

4.3 Discussion

The results of our power allocation scheme are shown in Fig. 3. As mentioned earlier, our erfc(z) approximation is

optimized for medium and high SNR values, therefore the resulting power allocation is not so effective at low uplink SNR values (0-4dB) as it is at medium to high values. Since our cooperation scheme uses M frames to fully complete, we compare our power allocation performance with a direct channel block coded system transmitting at M times the power over a single time frame. This will be referred to as a *No Cooperation*. The red squares show simulation results using the optimal power allocation factors determined from the optimization process. The corresponding red curve shows the theoretical equation (16). We show that the power allocation improves the BER compared with two other schemes using the same power budget (15): blue shows equal power allocation ($\alpha = \beta = 0.5$); and green shows power allocation optimizing a fixed fraction of power amongst the relays, which was proposed by Luo *et al.* [7]. The black line shows a non-cooperative scheme that uses $M-1$ times extra power. The magenta crosses show an alternative deterministic method of our proposed power allocation based upon (18).

4.4 Selfish Power Allocation

We now look at the impact of power allocation by comparing a $M = 2$ user unselfish and mutually selfish network. As discussed earlier, the chance of cooperation between two users in mutually selfish network is P_C^2, which is considerably less than the unselfish case that has a value of P_C. Otherwise, the theoretical BER and power constraint equation is very similar in both cases. Using a similar equation to (16), we perform the same analysis and found optimal power allocation and compared their performances. Fig. 4 shows the gain
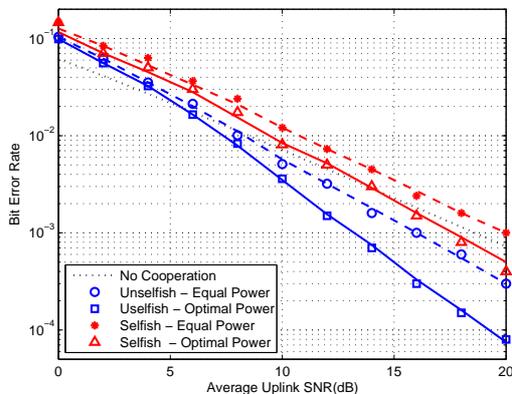
Figure 4: Comparing Power Allocation Benefit in Unselfish and Selfish Cooperation, for $M = 2$ users and 10dB Interuser Average SNR Environment

achieved by power allocation for both schemes. We see that the performance of the selfish system is worse than that of the unselfish system, because cooperation of one user relies on another. This constrains the cooperative power allocation to the far lower probability of P_C^2.

5. CONCLUSION

We began by finding approximated closed form expressions for the BER and FER of perfect block codes in block faded channels. This was extended to decode-and-forward cooperative environments, and we analyzed the BER performance difference of various systems. After which, we optimized the power allocation for the aforementioned scenarios using an iterative optimization scheme. An alternative simpler deterministic power allocation estimation technique has been obtained by differentiating the lower bound BER expression and it is shown to offer reasonable performance. Under a fixed long term power budget for each user, we have shown that an adaptive power allocation can improve the system performance over existing power allocation schemes. The proposed approach can be extended to other codes and DF networks.

6. REFERENCES

[1] M. Bossert. *Channel Coding for Telecommunications*. Wiley, London, UK, 1999.

[2] M. Chiani, D. Dardari, and M. K. Simon. New exponential bounds and approximations for the computation of error probability in fading channels. *IEEE Trans. on Wireless Communications*, 2(4):840–45, July 2003.

[3] X. Deng and A. Haimovich. Power allocation for cooperative relaying in wireless networks. *IEEE Trans. on Communications*, 50(12):3062–3080, July 2005.

[4] A. Goldsmith. *Wireless Communications*. Cambridge Uni. Press, Cambridge, UK, 2005.

[5] T. E. Hunter and A. Nosratinia. Coded cooperation under slow fading, fast fading, and power control. In *36th Asilomar Conference on Signals, Systems and Computers*, pages 118–22, Nov. 2002.

[6] J. N. Laneman, G. W. Wornell, and D. N. C. Tse. An efficient protocol for realizing cooperative diversity in wireless networks. In *Proc. Int. Symp. Information Theory*, page 294, Washington, USA, June 2001.

[7] J. Luo, R. Blum, L. Cimini, L. Greenstein, and A. Haimovich. Decode and forward cooperative diversity with power allocation. *IEEE Trans. on Inform. Theory*, 50(12):3062–80, Dec. 2004.

[8] I. Maric and R. Yates. Forwarding strategies for gaussian parallel relay networks. In *Proc. Int. Symp. Information Theory*, page 269, June 2004.

[9] A. Nosratinia, T. E. Hunter, and A. Hedayat. Cooperative communication in wireless networks. *IEEE Trans. on Wireless Communications*, 42(10):74–80, Oct. 2004.

[10] J. P. Odenwalder. Error control coding handbook. Tech. Rep. 60-92, LINKBIT, Aug. 1976.

[11] A. Ribeiro, X. Cai, and G. Giannakis. Symbol error probabilities for general cooperative links. *36th Asilomar Conference on Signals, Systems and Computers*, 4(3):1264–73, May 2005.

[12] G. Stuber. *Principles of Mobile Communication*. Kulwer Academic Pub., Dordrecht, NL, 1958.

[13] J. Wozencraft. *Principles of Communication Engineering*. Wiley, London, UK, 1965.

[14] Y. Zhao, R. Adve, and T. Lim. Improving amplify-and-forward relay networks: Optimal power allocation versus selection. *IEEE Trans. on Wireless Communications*, 6(8):3114–23, Aug. 2007.